Family Time-Income Allocation

Jose A. Rodrigues-Neto **Rongzhao Zhu**

Research School of Economics, Australian National University

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Motivation

Empirical studies confirm that rising spousal earning capacity sometimes destabilizes families:

- A higher earning capacity of the wife is likely to be related to a lower satisfaction of the husband and a higher probability of divorce (Bertrand et al. 2015, Folke & Rickne 2020)
- A husband is likely to use violence as an instrument to extract more surplus by damaging the wife's earning capacity (Bulte & Lensink 2021, Guarnieri & Rainer 2021)

Anderberg & Rainer (2013) studied a **non-cooperative** family model: There is a non-empty set of parameters in which one agent's equilibrium utility decreases with the partner's wage rate

This Paper

This paper models the decisions of a two-agent family with

- altruistic preferences 💞
- household public good and private consumption
- heterogeneous abilities of agents

We consider the family members' time and income allocation problem in two aspects:

(1) Non-cooperative family: Non-cooperative, simultaneous-move game $\rightarrow$ Nash equilibrium (NE)

(2) Cooperative family: Symmetric Nash bargaining using the NE allocation as a disagreement point

Main result: In a **cooperative** family, for each agent i, there is a non-empty set of parameters in which agent i's utility decreases with the partner $-i$'s wage rate

Setup: Agents and preferences

Two agents, are partners:

- Each agent i gets utility directly from the household public good H, and the private consumption c_i

$$u_i = \ln H + \ln c_i$$

- Altruistic preference

$$U_i = u_i + \alpha u_{-i}$$

  where $0 \leq \alpha \leq 1$ and $-i \in \{1, 2\}$ is the partner of i

- Payoff of agent i

$$U_i = \ln H + \ln c_i + \alpha \left(\ln H + \ln c_{-i} \right)$$

Setup: Time allocation

Each agent has one unit of time endowment

- $0 \leq x_i \leq 1$ housework
- $0 \leq 1 - x_i \leq 1$ market work
- A **time allocation** is a pair $x := (x_1, x_2)$

Setup: Public good, income and transfer

Given $x = (x_1, x_2)$

- Household public good H is home-produced

$$H(x) = a_1 x_1 + a_2 x_2$$

  - $a_i > 0$: agent i's ability of household production (exogenous)
- Agent i earns the wage income from the labour market

$$m_i(x_i) = (1 - x_i) w_i$$

  - $w_i > 0$: agent i's wage rate in labour market (exogenous)
- Total income M

$$M(x) = (1 - x_1) w_1 + (1 - x_2) w_2$$

Each agent can make a monetary transfer τ_i to the other

- $0 \leq \tau_i \leq m_i(x_i)$: the transfer from agent i to $-i$

Setup: Private consumption and income allocation

The private consumption $=$ the income after transfer

- $c_i(x, \tau) = (1 - x_i) w_i - \tau_i + \tau_{-i}$

Let $\rho_i \in (0, 1)$ be the proportion of agent i's private consumption to the total income

$$\rho_i := \frac{c_i}{M}$$

- $\rho_1 + \rho_2 = 1$ as $c_1 + c_2 = M$
- An **income allocation** is a pair $\rho := (\rho_1, \rho_2)$

Payoff of agent i

$$U_i(x, \tau) = (1 + \alpha) \ln H + \ln c_i + \alpha \ln c_{-i} \quad \text{or equivalently,}$$
$$U_i(x, \rho) = (1 + \alpha) \ln H + \ln(\rho_i M) + \alpha \ln(\rho_{-i} M)$$

Assumption

Assumption 1.

The parameters a_1, w_1, a_2, w_2 and α are common knowledge and satisfy to

$$\frac{1}{2+\alpha} a_1 \leq a_2 \leq a_1$$
$$\alpha w_2 \leq w_1 \leq w_2$$

The assumption implies $\frac{w_2}{a_2} \geq \frac{w_1}{a_1}$:

- Agent 1 has both the absolute advantage and the comparative advantage in household production
- Agent 2 has both the absolute advantage and the comparative advantage in earning

Non-cooperative Family

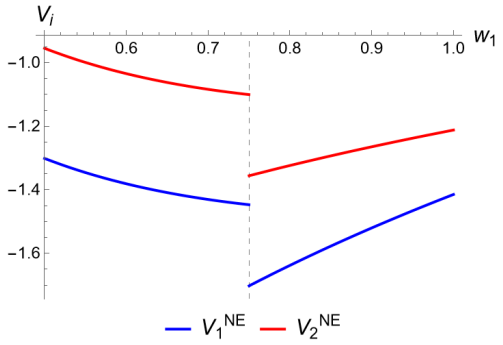
In a non-cooperative family, agents simultaneously choose (x_i, τ_i)

(1) If $w_1 \leq \alpha \frac{a_2}{a_1} w_2$, then $\frac{\partial V_1^{NE}}{\partial w_1} = \frac{\partial V_2^{NE}}{\partial w_1} < 0$

- Agent 2 transfers money to agent 1
- The transfer
  - supports agent 1's private consumption
  - influences agent 1's time allocation away from market work
- A higher wage rate of agent 1 means
  - more labour supply from agent 1
  - less household production
  - less specialization
- Similar to Anderberg & Rainer (2013)

(2) If $w_1 > \alpha \frac{a_2}{a_1} w_2$, then $\frac{\partial V_1^{NE}}{\partial w_1} > \frac{\partial V_2^{NE}}{\partial w_1} > 0$

- There is no transfer in NE
- The time allocation is independent of (w_1, w_2)



Pareto Efficiency

Proposition 1 (Pareto efficient allocation)

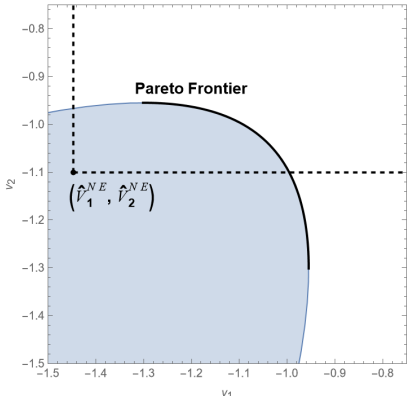
Suppose **Assumption 1** holds, i.e. $\frac{1}{2+\alpha} a_1 \leq a_2 \leq a_1$ and $\alpha w_2 \leq w_1 \leq w_2$.

An allocation $\left(x^{PE}, \rho\right)$ is Pareto efficient iff $\left(x_1^{PE}, x_2^{PE}\right) = (1, 0)$ and $\rho_i \in \left[\frac{\alpha}{1+\alpha}, \frac{1}{1+\alpha}\right]$ for $i \in \{1, 2\}$.

Pareto frontier:

$$\left\{ (v_1, v_2) \middle| \begin{array}{l} v_1 = (1+\alpha)\ln(a_1 w_2) + \ln\rho_1 + \alpha\ln\rho_2, \\ v_2 = (1+\alpha)\ln(a_1 w_2) + \ln\rho_2 + \alpha\ln\rho_1, \\ \rho_2 = 1 - \rho_1 \text{ and } \rho_1 \in \left[\frac{\alpha}{1+\alpha}, \frac{1}{1+\alpha}\right] \end{array} \right\}$$

- The change of w_1 does **not** affect the Pareto frontier
- The NE allocation is Pareto **in**efficient



Cooperative Family's Bargaining Problem

In a cooperative family, agents negotiate the income allocation ρ

- The disagreement point V_i^{NE} is agent i's payoff from the NE
- The maximisation problem is

$$\begin{aligned} \max_{\rho} \quad & \left(U_1\left(x^{PE}, \rho\right) - V_1^{NE}\right)\left(U_2\left(x^{PE}, \rho\right) - V_2^{NE}\right) \\ \text{s.t.} \quad & U_i\left(x^{PE}, \rho\right) \geq V_i^{NE} \quad \forall i \in \{1, 2\} \end{aligned} \tag{1}$$

We show that the cooperative family's problem (1) has a unique solution

- Let V_i^C be agent i's payoff in a cooperative family

Main Result: Conflict of Interest

Proposition 2

Suppose **Assumption 1** holds, i.e. $\frac{1}{2+\alpha} a_1 \leq a_2 \leq a_1$ and $\alpha w_2 \leq w_1 \leq w_2$.
Then, V_1^C strictly **increases** with w_1 and V_2^C strictly **decreases** with w_1.

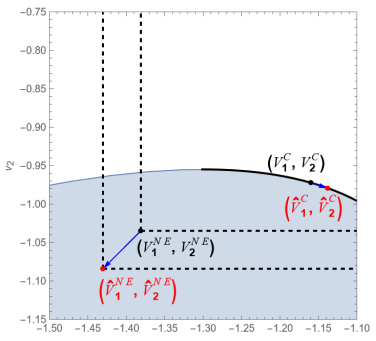
Suppose w_1 increases marginally:
$w_1 \nearrow \hat{w}_1$

(a) $w_1 \leq \hat{w}_1 < \alpha \frac{a_2}{a_1} w_2$:

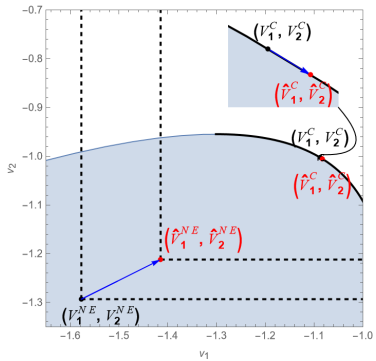
- $V_1^{NE} \searrow \hat{V}_1^{NE}$ and $V_2^{NE} \searrow \hat{V}_2^{NE}$
- $V_1^C \nearrow \hat{V}_1^C$ and $V_2^C \searrow \hat{V}_2^C$

(b) $\alpha \frac{a_2}{a_1} w_2 < w_1 \leq \hat{w}_1$:

- $V_1^{NE} \nearrow \hat{V}_1^{NE}$ and $V_2^{NE} \nearrow \hat{V}_2^{NE}$
- $V_1^C \nearrow \hat{V}_1^C$ and $V_2^C \searrow \hat{V}_2^C$

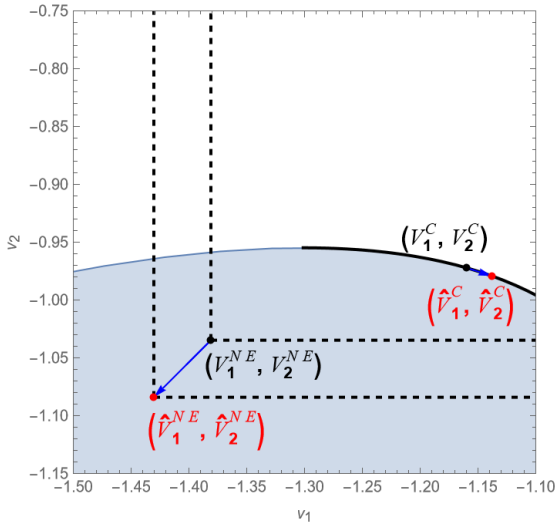


(a) $w_1 \leq \hat{w}_1 < \alpha \frac{a_2}{a_1} w_2$

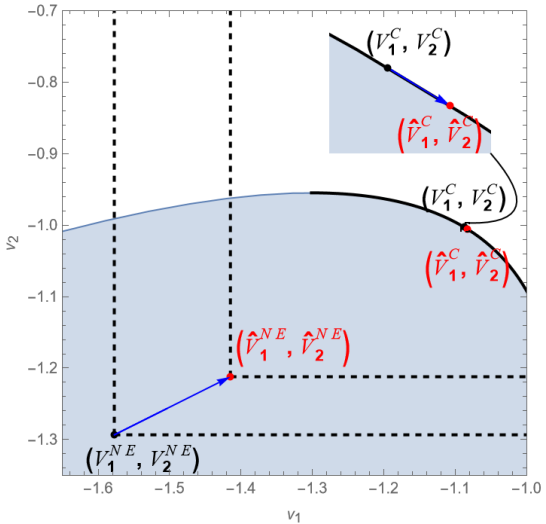


(b) $\alpha \frac{a_2}{a_1} w_2 \leq w_1 < \hat{w}_1$

Conflict of Interest in Cooperative Family



(a) $w_1 \leq \hat{w}_1 < \alpha \frac{a_2}{a_1} w_2$



(b) $\alpha \frac{a_2}{a_1} w_2 \leq w_1 < \hat{w}_1$

Conclusion

Based on Nash equilibrium for non-cooperative families and Pareto efficient allocations, we analyse cooperative families choosing an allocation by symmetric Nash bargaining, using the non-cooperative NE as the disagreement point

- In a cooperative family, each agent is strictly worse off due to an increase in the partner's wage rate if the agent has both comparative and absolute advantages in earning, but the advantages are not strong enough to induce a Pareto-efficient NE
- The degree of altruism matters. A higher degree of altruism means agents' welfare is more likely to increase with their partner's wage rate.

The End

Thanks!

Related Literature

Empirical reasearch

- Marriage stability: Bertrand et al. (2015), Folke & Rickne (2020)
- Domestic violence: Bulte & Lensink (2021), Guarnieri & Rainer (2021)

Models for household decision making

- Unitary approach: Becker (1974)
- Non-cooperative approach: Leuthold (1968), Konrad & Lommerud (1995, 2000), Lechene & Preston (2011), Anderberg & Rainer (2013), Del Boca & Flinn (2012, 2014)
- Cooperative approach
  - Collective model: Chiappori (1988, 1992, 1997), Browning & Chiappori (1998), Chiappori (2011), Chiappori et al. (2018)
  - Nash bargaining: Manser & Brown (1980), McElroy & Horney (1981), Lundberg & Pollak (1993), Chen & Woolley (2001), Cherchye et al. (2013), Cohen & Glazer (2017), Yakita (2018), McPeak & Doss (2006), Walther (2018), Browning et al. (2010)

General Conclusion

Assume $\frac{1}{2+\alpha} \leq \frac{a_2}{a_1} \leq 1$. This assumption rules out the 'free-riding' NE.

In a non-cooperative family

- the allocation is Pareto efficient if the wage gap is sufficiently large
- there is a non-empty interval of agent 1 's wage rate such that agent 2 's welfare is strictly decreasing with agent 1's wage rate
- both agents' welfare jumps down if the equilibrium changes from a positive-transfer NE to a zero-transfer NE due to some small change of w_1 or w_2

In a cooperative family

- agents are weakly better off due to an increase of their own wage rate
- agents are strictly worse off due to an increase of the partner's wage rate if
  - the agent has both comparative and absolute advantages in the labour market, and the advantages are not strong enough to induce a Pareto-efficient NE
  - the agent has the comparative but no absolute advantages in the labour market, and the altruism is low

General Conclusion

The degree of altruism matters. A higher degree of love means

- the non-cooperative allocation is more likely to be Pareto efficient;
- agents' welfare is more likely to be increasing with their partner's wage rate, in both non-cooperative families and cooperative families.

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